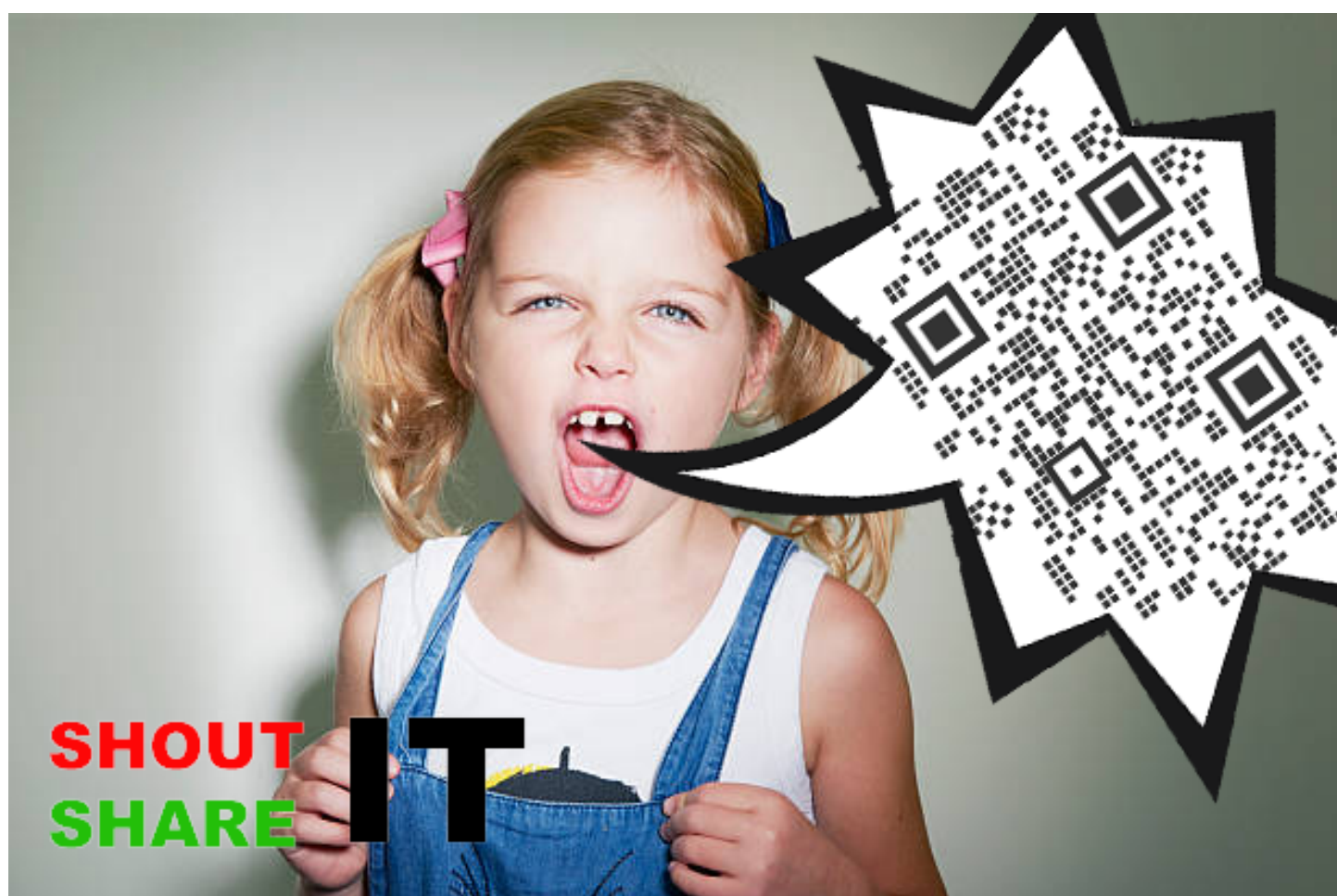


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WEIGHT VARIATION OF PACKAGE FOODS.

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INTRODUCTION.

The food official and the food packer often ask, "What is the variation in weight of package foods?" The answer might be simple when only a particular product packed at a particular factory is considered, but the complex manipulations in packing and the range of supervision employed at different factories make any answer subject to qualifications when applied to an entire industry such as the packing of spices, coffee, sugar, rice, and similar free-flowing substances. The establishments which pack these products, however, have some methods of manipulation in common, which, when given reasonably adequate supervision, result in a fairly definite range of variation in weight. It is recognized that an answer might be obtained by different methods, but the steps by which an answer has been reached in this bulletin are (1) selecting specifications for a method of weighing to represent good commercial practice in packing granular, free-flowing products, (2) calculating the maximum errors expected from these specifications, and (3) showing that the calculated errors approximate the variations in weight found by the commercial application of the specified method.

In accordance with the phraseology of the theory of errors which has been employed in the calculations, variations from the intended weight are called "errors."

¹ Acknowledgment is made to J. C. Munch, of the Bureau of Chemistry, for assistance in the collection and compilation of the data, and to the various members of the field force of the Bureau of Chemistry who assisted in the collection of information from various factories.

METHODS OF WEIGHING.

From information obtained from 126 factories located in 21 different States, it was found that the methods of weighing fall fairly well into the following five general classes:

A. Hand weighed:

1. Contents of each package weighed net before being packaged.
2. Each package tare weighed, filled, and adjusted to the proper gross weight.

B. Machine weighed:

1. Contents of each package weighed net before being packaged.
2. Average tare assumed, package filled, and adjusted to the proper gross weight.
3. Weight of contents estimated by the size of each container.

Of the weighing methods found for packages exceeding 5 pounds, 95 per cent were hand weighed (A), and of these 88 per cent were gross and tare weighed (A2). Its exceptional prevalence and correctness of principle recommend this method (A2) as representative of good commercial practice for packages of these sizes.

For packages of 5 pounds and under about half were hand weighed (A) and about half machine weighed (B). Of the hand weighed about half were weighed net (A1), and about half gross and tare (A2); but of the machine weighed, about 95 per cent were weighed net (B1). The importance of hand weighing is emphasized by its applicability to all types of products, its equal prevalence, and its priority in the field. The importance of hand weighing net (A1) is emphasized by its equal prevalence, its correctness of principle, and by the fact that no new machinery is required for its adoption. Its importance recommends hand weighing net (A1) as a representative method of good commercial practice on sizes of 5 pounds and below.

GOOD COMMERCIAL PRACTICE OF HAND WEIGHING.

Supplemental investigations showed that unwarranted variations occur in hand-weighed packages because of indifference to the type, capacity, repair, and manipulation of scales. Observations were made on all the scales in use at each of the 126 factories inspected.

For packages of 5 pounds and below, even arm balances with no beam were found in 41 per cent of the instances. Even arm balances carrying a 1-pound beam and graduated to one-fourth ounce were found in 37 per cent of the instances. The relative prevalence and adaptability to the packer's needs of the beam scale favor its selection as a type to represent good commercial practice for these sizes.

For packages of 25 pounds and over, a platform scale with a 50-pound beam graduated to one-fourth pound was found in about 50 per cent of the instances. The relative prevalence and commercial adaptability of this scale favor its selection as a type to represent good commercial practice for these sizes.

In 68 per cent of all the instances observed, the scales were used at 20 per cent or more of their capacity. For packages of below 25 pounds such a capacity is quite adaptable and commercially available, but for those of larger size a 400- to 600-pound scale is much more suitable to the packer's general needs and more available on the market. Practicability and prevalence of the practices favor the selection of 20 per cent of the capacity on smaller sizes and 10 per cent for the larger sizes as the minimum weight which may be weighed on scales to represent good commercial practice. Tare weights are usually made on the same scale as the gross weights.

Adoption of tolerances on scales by the Bureau of Standards and the Annual Conferences of Weights and Measures Officials¹ favors the selection of these tolerances as a control on the condition of scales to represent good commercial practice.

Inability to determine an exact balance and to set the poise exactly on a graduation are the principal errors in manipulating scales. In a supplemental experiment an inspector and two commercial packers weighed the same 50 packages under commercial conditions on a scale having a known sensibility reciprocal (weight required to move scale beam from exact balance to the limit of its motion). Only 9 of the 150 weights were in error by more than one-fifth of the sensibility reciprocal. The error in the manipulation of a scale in good condition is limited to the total sensibility reciprocal. No apparent hardship, therefore, would be exacted of the packer if one-half of the sensibility reciprocal were selected to represent good commercial practice as to this manipulation.

Since the poise is commonly set with greater care than is used in estimating the equilibrium, and since the operation is very simple (setting a point on the poise to meet a line on the beam), no apparent hardship would be exacted of the packer if one-fourth of a minimum graduation were selected as the limit of error to represent good commercial practice in this manipulation.

Accordingly, the following specifications have been selected to represent good commercial practice in packaging granular free-flowing products, such as tea, coffee, spices, etc.:

1. That packages of 5 pounds and below be weighed net; larger packages may be weighed gross and tare.
2. That the following types of scales be used—
 - For packages of 5 pounds and below: Even arm balance, 1-pound beam, one-fourth ounce graduation.
 - For packages of over 5 and under 25 pounds: Even arm balance, 2-pound beam, one-fourth ounce graduation.
 - For packages of 25 pounds and over: Platform scale, 50-pound beam, one-fourth pound graduation.

¹ U. S. Dept. Commerce, Bur. Standards Circ. 61.

3. That scales meet the tolerances prescribed in Bureau of Standards Circular 61 and be used at 20 per cent or more of their capacity for packages of less than 25 pounds, and at 10 per cent or more of their capacity for larger packages. Tare weights may be made on the same scale as the gross weights.
4. That the exact balance be estimated to within one-half of the sensibility reciprocal of the scale, and the poise be set to within one-fourth of the minimum graduation on the beam.

These specifications are drawn up in this form for the particular purpose of having something specific on which to base calculations. The study of factories has shown them to be in general use.

CALCULATED MAXIMUM ERRORS OF GOOD COMMERCIAL PRACTICE.

The calculation of the maximum errors of good commercial practice is based on the law of propagation of errors, the Bureau of Standards tolerances on scales, and the specifications of good commercial practice.

The law of propagation of errors, as found in standard text books¹ on the methods of least squares, is—

$$E = \sqrt{(a)^2 + (b)^2 + (c)^2 + (d)^2},$$

where (a) , (b) , (c) , and (d) are constituent errors which combine to form the resultant error E , all errors having the same probability of occurrence and the same chance of being plus or minus.

The Bureau of Standards tolerances on scales, as found in Circular 61 of that Bureau, are given in Tables 1, 2, 3, and 4. A general attempt is being made by weights and measures officials to eliminate all scales which fail to meet these tolerances, thereby leaving in general use scales which do meet them.

The specifications of good commercial practice (p. 3) designate the figures to be selected from the tolerance tables (constituent errors), which in turn are combined according to the law of propagation of errors to give the resultant calculated maximum error. This calculated maximum error will have the same probability of occurrence as the constituent errors. It is scarcely conceivable that all the various tolerances have the same probability of occurrence, but their long use by weights and measures officials for the purpose of condemning scales makes them a stable basis of a limiting probability. Other figures, such as one-fourth of a minimum graduation, used in determining constituent errors, are assumed to have probabilities of occurrence about equivalent to the tolerance figures. No attempt has been made to weight the constituent errors in order to correct for their unequal probabilities. The calculated maximum error, there-

¹ Merriman, Mansfield. A Text-Book on the Method of Least Squares, 8th ed. New York, 1913.

TABLE 1.—*Tolerances allowable on counter scales (even arm and other types).*¹

Load.	Tolerance on parts requiring employment of removable weights.	Tolerance on beam or reading face.	Load.	Tolerance on parts requiring employment of removable weights.	Tolerance on beam or reading face.
Pounds.	Ounce.	Ounce.	Pounds.	Ounces.	Ounces.
1	$\frac{1}{16}$	$\frac{1}{16}$	40	$\frac{1}{16}$	$\frac{3}{4}$
2	$\frac{1}{8}$	$\frac{1}{8}$	50	$\frac{1}{8}$	$\frac{3}{4}$
4	$\frac{1}{4}$	$\frac{1}{4}$	60	$\frac{1}{4}$	1
5	$\frac{1}{5}$	$\frac{1}{5}$	75	$\frac{1}{5}$	1
6	$\frac{1}{6}$	$\frac{1}{6}$	90	$\frac{1}{6}$	$1\frac{1}{2}$
8	$\frac{1}{8}$	$\frac{1}{8}$	100	1	$1\frac{1}{2}$
10	$\frac{1}{10}$	$\frac{1}{10}$	150	$1\frac{1}{2}$	2
12	$\frac{1}{12}$	$\frac{1}{12}$	200	2	3
15	$\frac{1}{15}$	$\frac{1}{15}$	240	$2\frac{1}{2}$	4
16	$\frac{1}{16}$	$\frac{1}{16}$	250	$2\frac{1}{2}$	4
20	$\frac{1}{10}$	$\frac{1}{10}$	300	3	$4\frac{1}{2}$
24	$\frac{1}{12}$	$\frac{1}{12}$	350	$3\frac{1}{2}$	5
25	$\frac{1}{10}$	$\frac{1}{10}$	400	4	6
30	$\frac{1}{10}$	$\frac{1}{10}$			

TABLE 2.—*Maximum sensibility reciprocal allowable on counter scales (even arm and other types).*¹

Capacity.	Maximum sensibility reciprocal allowable. ²	Capacity.	Maximum sensibility reciprocal allowable. ²
Pounds.	Ounce.	Pounds.	Ounces.
1	$\frac{1}{4}$	24	1
2	$\frac{1}{2}$	25	1
4	1	30	1
5	$1\frac{1}{4}$	40	$1\frac{1}{4}$
6	$1\frac{1}{2}$	50	$1\frac{1}{2}$
8	$1\frac{2}{3}$	60	$1\frac{2}{3}$
10	2	75	2
12	$2\frac{1}{2}$	90	$2\frac{1}{2}$
15	3	100	3
20	4		

TABLE 3.—*Tolerances allowable on platform scales used indoors.*¹

Load.	Tolerance, Class A. ³	
	On ratio.	On beam.
Pounds.	Ounces.	Ounces.
50	$\frac{1}{2}$	1
100	1	2
200	2	4
240	3	6
300	3	6
400	4	8
500	5	10
600	6	12
800	8	1
1,000	8	1
1,200	10	$1\frac{1}{2}$

TABLE 4.—*Tolerances allowable on all counterpoise weights (avoirdupois system).*¹

Weight.	Tolerance, ordinary weights (ratio 1:1).	Tolerance, counterpoise weights for multiplying-lever scales.			Weight.	Tolerance, ordinary weights (ratio 1:1).	Tolerance, counterpoise weights for multiplying-lever scales.		
		Ratio less than 100:1.	Ratio 100:1 and less than 1,000:1.	Ratio 1,000:1 and over.			Ratio less than 100:1.	Ratio 100:1 and less than 1,000:1.	Ratio 1,000:1 and over.
Pounds.	Grains.	Grains.	Grains.	Grains.	Ounces.	Grains.	Grains.	Grains.	Grains.
50	100.0	60.0	40.0	20.0	10	10.0	6.0	4.0	2.0
25	60.0	36.0	24.0	12.0	8	5.0	3.0	2.0	1.0
20	60.0	36.0	24.0	12.0	5	5.0	3.0	2.0	1.0
15	40.0	24.0	16.0	8.0	4	5.0	3.0	2.0	1.0
10	40.0	24.0	16.0	8.0	2	3.0	1.8	1.2	.6
8	30.0	18.0	12.0	6.0	1	2.0	1.2	.8	.4
5	30.0	18.0	12.0	6.0		2.0	1.2	.8	.4
4	20.0	12.0	8.0	4.0		1.0	.6	.4	.2
3	20.0	12.0	8.0	4.0		.5	.3	.2	.1
2	15.0	9.0	6.0	3.0		$\frac{1}{8}$	.5	.3	.2
1	10.0	6.0	4.0	2.0		$\frac{1}{16}$	.5	.3	.2
						$\frac{1}{32}$	.2	.12	.08

¹ These tolerances are taken from U. S. Dept. Commerce, Bur. Standards Circ. 61.² Two minimum graduations shall be the tolerance on the sensibility reciprocal if such value is less than that given here.³ The sensibility reciprocal allowable is two minimum graduations of the scale in question.

fore, is defined for the purpose of this bulletin as that error in the weight of the finished package which has approximately the same probability of occurrence as each of the constituent errors.

As illustrations of the method of calculation, two examples, one of net weighing (3-pound packages) and one of gross and tare weighing (75-pound packages), are worked out in detail. The results on packages of other sizes are given in Tables 5 and 6. Such combinations of constituent errors as will result in the maximum error have been chosen.

3-POUND PACKAGES.

The specifications of good commercial practice require that a 3-pound package shall be weighed net on an even-arm scale of not more than 15 pounds capacity and carrying a 1-pound beam graduated to one-fourth ounce, and that the poise of this scale shall be set to within one-fourth of a minimum graduation, and that no error greater than one-half of the sensibility reciprocal be made in estimating the exact balance. Then the maximum constituent errors introduced, as read from the tolerance tables and specifications, are:

a. "On parts requiring the employment of removable weights" (Col. 2, Table 1) for 2 pounds.....	$\frac{1}{16}$ or 0.062 ounce.
b. "On beam or reading face" (Col. 3, Table 1) for 1 pound.....	$\frac{1}{16}$ or 0.062 ounce.
c. Counterpoise weights times ratio of scale, 2 pounds times 1 (Table 1).....	15 grains or 0.034 ounce.
d. One-fourth of a minimum graduation ($\frac{1}{4}$ of $\frac{1}{4}$ ounce)...	$\frac{1}{16}$ or 0.062 ounce.
e. One-half of the sensibility reciprocal (Table 2) for 15 pounds ($\frac{1}{2}$ of $\frac{1}{2}$ ounce).....	$\frac{1}{4}$ or 0.250 ounce.

For all practical purposes errors *a*, *b*, *c*, and *d* have approximately the same probability of occurrence, which is quite small, since they are the maximum tolerances allowable, and may be either plus or minus in any particular scale. Consequently, the law of propagation of errors may be applied, and $\sqrt{(a)^2 + (b)^2 + (c)^2 + (d)^2}$ represents the calculated maximum constant error introduced into every package by a scale, the error of which is just within the tolerance after the most inaccurate attempt permitted has been made to set it on the proper graduation.

Error *e* is variable for every package, and is directly additive to the calculated maximum constant error. Then the calculated maximum error on any single package will be—

$$\sqrt{(0.062)^2 + (0.062)^2 + (0.034)^2 + (0.062)^2} + (0.250) \text{ or } 0.363 \text{ ounce.}$$

According to the laws of errors the error on the average of a random selection from a total distribution is $\frac{\text{Variable Error}}{\sqrt{n}}$, where

n is the number selected. In this case the resulting error has the same probability of occurrence as the variable error. Error *e* is the maxi-

imum variable error allowable by the specifications. The value selected for n is 50, since an average shipment usually contains at least 50 packages, and experience has shown that 50 is a sufficiently representative sample.

Then the calculated maximum error on the average weight of a representative sample of 3-pound packages is—

$$\sqrt{(0.062)^2 + (0.062)^2 + (0.034)^2 + (0.062)^2} + \frac{0.250}{\sqrt{50}}, \text{ or } 0.148 \text{ ounce.}$$

75-POUND PACKAGES.

The specifications of good commercial practice require that a 75-pound package shall be weighed gross and tare on a platform scale of not more than 750 pounds capacity, carrying a 50-pound beam graduated to one-fourth pound, and that the poise of this scale be set to within one-fourth of a minimum graduation, and that no error greater than one-half the sensibility reciprocal be made in estimating the exact balance.

Then the maximum constituent errors as read from the tolerance tables and specifications are:

Constant errors introduced into every package:

	Ounces.
On tares (assumed to be 15 pounds)—	
a. On ratio.....	0.00
b. On beam (Col. 3, Table 3) for 50 pounds or less.....	1.00
c. Counterpoise weights times ratio of scale.....	.00
On gross (net plus tare: 75 + 15, or 90 pounds)—	
d. On ratio (Col. 2, Table 3) for 50 pounds.....	.50
e. On beam (Col. 3, Table 3) for 50 pounds or less.....	1.00
f. Counterpoise weights times ratio of scale (Col. 3, Table 4) for 50-pound counterpoise, which is an 8-ounce weight on a ratio of 100 : 1, or 3 grains times 100.....	.69

Variable errors due to the manipulation of the scale:

On tares—	
g. Since the weight of the tare can not be changed, it will be weighed to the nearest graduation of the scale, and the reading may be in error by one-half the minimum graduation ($\frac{1}{2}$ of 4 ounces).....	2.00
h. One-half of the sensibility reciprocal (one-half of twice the minimum graduation (Table 3)) ($\frac{1}{2}$ of 8 ounces).....	4.00
On gross—	
i. One-fourth of a minimum graduation for setting of poise.....	1.00
k. One-half of the sensibility reciprocal (Table 3) ($\frac{1}{2}$ of 8 ounces).....	4.00

The constituent constant errors and the constituent variable errors will each combine according to the law of propagation of errors as in the case of 3-pound packages, and the calculated maximum error on any single package will be given by the direct sum of the two results, that is—

$$\sqrt{(a)^2 + (b)^2 + (c)^2 + (d)^2 + (e)^2 + (f)^2} + \sqrt{(g)^2 + (h)^2 + (i)^2 + (k)^2}$$

or

$$\sqrt{(0.00)^2 + (1.00)^2 + (0.00)^2 + (.50)^2 + (1.00)^2 + (.69)^2} + \sqrt{(2.00)^2 + (4.00)^2 + (1.00)^2 + (4.00)^2},$$

which is 7.73 ounces.

TABLE 6.—*Calculated maximum errors of good commercial practice (gross and tare weighing).*

Size of package.	Scales specified.					Probable maximum weight of tares.	Parts of scale used for calculation.				Average number packages in representative sample.				
	Maximum capacity.	Capacity of beam.	Minimum graduation.	Type.	Ratio.		On tares.		On gross.						
							Lbs.	Ozs.	Lbs.	Ozs.		Weight.	Beam.		
														Lbs.	Ozs.
Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.								
10.....	50	2	$\frac{1}{4}$	E. A.	1:1	1	0	1	9	2	10				
25.....	250	50	4	Ptf.	100:1	5	0	5	0	30	3				
50.....	500	50	4	Ptf.	100:1	10	0	10	50	10	3				
75.....	750	50	4	Ptf.	100:1	15	0	15	50	40	1				
100 and above.....	1,000	50	4	Ptf.	100:1	20	0	20	100	20	1				
Maximum constituent errors introduced as read from tolerance tables and specifications.															
Size of package.	Constant errors introduced into every package.					Variable errors due to manipulation of scale.									
	On gross.					On tares.					On gross.				
	On ratio.	On beam.	Counterpoise weights times ratio of scale.	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(k)				
												Counterpoise weights times ratio of scale.	(c)	(d)	(e)
Oz.	Oz.	Oz.	Oz.	Oz.	Oz.	Oz.	Oz.	Oz.	Oz.	Oz.					
10.....	0	0.250	0	0.062	0.125	2	0.069	0.125	0.250	0.062	0.250	0.414			
25.....	0	1.00	0	0.06	1.00	2.00	0.00	2.00	4.00	1.00	4.00	3.34			
50.....	0	1.00	0	0.50	1.00	3.00	0.09	2.00	4.00	1.00	4.00	3.57			
75.....	0	1.00	0	0.50	1.00	3.69	0	2.00	4.00	1.00	4.00	3.57			
100 and above.....	0	1.00	0	1.00	1.00	4.137	0	2.00	4.00	1.00	4.00	4.13			
$\sqrt{(a)^2 + (b)^2 + (c)^2 + (d)^2 + (e)^2 + (f)^2} + \sqrt{(g)^2 + (h)^2 + (i)^2 + (k)^2} \div \sqrt{n}$															
Calculated maximum error on the average weight of a representative sample,															

¹ Smallest tolerance given in tables.² Tolerance on 8-pound weights used.³ The 50-pound weight used on a scale of 100:1 ratio weighs 8 ounces.⁴ The 100-pound weight used on a scale of 100:1 ratio weighs 1 pound. from the table in the column ratio less than 100:1 and multiplied by 100.

In actual practice scales for this purpose are rarely above 100:1 ratio. For this reason tolerances are taken

Since an average shipment contains only 1 package, it would be difficult to select 50 in actual practice. Hence, 10 is used as a representative sample. Then the calculated maximum error on the average weight of a representative sample is—

$$\frac{\sqrt{(0.00)^2 + (1.00)^2 + (0.00)^2 + (.50)^2 + (1.00)^2 + (.69)^2 + (2.00)^2 + (4.00)^2 + (1.00)^2 + (4.00)^2}}{\sqrt{10}},$$

or 3.57 ounces.

The same general scheme of calculation has been followed in Tables 5 and 6. The average number of packages per shipment was determined by inquiry at a large number of factories and inserted in round numbers. Any departure from the rules laid down are explained in the footnotes of the tables.

EXPERIMENTS ON GOOD COMMERCIAL PRACTICE METHODS.

In order to try out the specified method of packing, a series of experiments was conducted in several factories. The method of packing was outlined to the packer, unless he was using it already, and care was exercised to see that all details were complied with and that a commercial speed was maintained. When everything was in good working order and the actual aim of the packer as to the weight he intended to put in the package had been determined, 50 packages were put up by him, after which they were carefully reweighed on an accurate scale to determine the actual variations. The results of these experiments are given in Table 7.

Column 6 shows that some results are plus and some minus, that the averages of the various groups are near 0, and that more results are near their group averages than are far away from them. Practically the same conditions govern normal frequency distributions; that is, if an infinite number of results are taken when the desired average minus aim is 0, the number of pluses and minuses will be approximately the same, the average will be 0, and comparatively few results will be far from 0. Then the groups are random selections from normal frequency distributions, the precision measures of which are the figures under the designation, "Average deviation from 0 average minus aim," given in the last column of the table. For purposes of comparison the practical limit of distribution of each group is considered to be the figure for the corresponding size given in the last column of Table 5 and of Table 6, headed "Calculated maximum error on the average of a representative sample."

It has been found that the weights of single packages taken from a single factory at one time follow a normal frequency distribution. The precision measure of each distribution of single packages from their averages is given by the figures in the last column of Table 7, "Average deviation of single packages from average." For the sake

of comparison, the practical limit of each of these distributions is considered to be the corresponding figure in the next to the last column of Table 5 and of Table 6, "Calculated maximum error on a single package."

TABLE 7.—*Observed variations of good commercial practice.*

Experiment No.	Product.	Declared weight.	Aim. ¹	Average found.	Average minus aim.	Average deviation of single packages from average.
		Ounces.	Ounces.	Ounces.	Ounces.	Ounces.
860a.....	Cayenne pepper.....	0.75	0.78	0.76	—0.02	0.050
852b.....	Tea.....	1.33	1.40	1.40	.00	.031
852c.....	do.....	1.33	1.35	1.37	.02	.042
860b.....	Ginger.....	2.00	2.03	2.05	.02	.025
860d.....	Greek sage.....	2.00	2.02	2.04	.02	.024
860f.....	Ground mustard.....	2.00	2.05	2.04	— .01	.015
860g.....	Ginger.....	2.00	2.04	2.03	— .01	.025
Averages of approximately 2-ounce packages.....					.003	.030
Average deviation from 0 average minus aim.....						.014
860e.....	Pickling spice.....	3.50	3.55	3.56	.01	.031
853a.....	Cinnamon.....	4.00	4.05	4.12	.07	.018
854a.....	Ginger.....	4.00	4.07	4.09	.02	.026
854b.....	Cinnamon.....	4.00	4.05	4.08	.03	.017
Averages of approximately 4-ounce packages.....					.032	.023
Average deviation from 0 average minus aim.....						.032
851a.....	Dessert powder.....	7.00	7.05	7.08	.03	.027
851b.....	do.....	7.00	7.03	7.20	2.17	.061
851c.....	do.....	7.00	7.15	7.17	.02	.052
853b.....	Ground nutmeg.....	8.00	10.70	10.72	.02	.027
Averages of approximately 8-ounce packages.....					.023	.042
Average deviation from 0 average minus aim.....						.023
855.....	Pulverized coffee.....	1 lb.	16.07	16.07	.00	.024
858a.....	Rice.....	1 lb.	16.02	16.10	.08	.031
860c.....	Rice flour.....	1 lb.	16.04	16.07	.03	.064
863a.....	Coffee.....	1 lb.	16.00	16.02	.02	.036
Averages of 1-pound packages.....					.032	.039
Average deviation from 0 average minus aim.....						.032
857b.....	Coffee.....	3 lbs.	59.16	59.28	.12	.094
Average deviation from 0 average minus aim.....						.120
852a.....	Ground cloves.....	5 lbs.	79.55	79.55	.00	.120
861a.....	Baking powder.....	5 lbs.	89.46	89.45	— .01	.218
Averages of 5-pound packages.....					— .005	.169
Average deviation from 0 average minus aim.....						.005
854e ²	Pepper.....	50 lbs.	800.00	799.30	— .70	.400
Average deviation from 0 average minus aim.....						.70

¹ The operators were giving a slight overweight, the amount of which was estimated by observing the method of weighing and the sensibility reciprocal of the scale. Addition of this amount to the reading of the scale gave the aim.

² This operator persisted in giving excessive down weight, contrary to instructions. His results are not included in the average.

³ This set was weighed with exceptional care by the packer, but all other conditions were commercial.

COMPARISON OF CALCULATED RESULTS WITH OBSERVED RESULTS.

Although the theory of probability provides for errors of infinite magnitude from the average of a normal distribution, in practice their magnitude is limited. The limitations may be measured by the calculated chances of occurrence of the maximum error. A

study of 736 normal frequency distributions made by commercial packers on the weight of tea, coffee, cocoa, spices, butter, cheese, condensed milk, starch, flour, and various other food products, 50 observations being made on each distribution, showed that the most frequent chance of occurrence of the maximum error was 1 in 60; the average, 1 in 198; and that 80 per cent were 1 in 1,000 or less. Many of those greater than 1 in 1,000 were due to avoidable causes, such as inaccurate reweighing, and occasional slips of the packer.

The probability of occurrence of errors within a certain limit is found by the formula $f = \frac{x}{r}$, where f is a factor whose probability value is found from a table of probability integrals,¹ x is the limiting error whose probability is desired, and r is the probable error of a single observation of the distribution. The complement of the equivalent of f gives the probability that errors greater than x will occur, and the resulting fraction reduced until its numerator is 1 gives the chances of their occurrence.

In the case of average weights, x represents the calculated maximum error on the average of a representative sample, and r for practical purposes is 0.8453 times the average deviation from 0 average minus aim. The resulting chances are given in Table 8.

TABLE 8.—*Chance of occurrence of variations greater than the calculated maximum error on the average of a representative sample.*

Size of package.	Calculated maximum error on average of a representative sample (x).	Average deviation from 0 average minus aim.	0.8453 $\times$ average deviation from 0 average minus aim. (r).	$\frac{x}{r}$ (f).	Chance of occurrence of variations greater than calculated maximum, based on observed data.
	Ozs.	Oz.	Oz.		
2-oz.....	0.097	0.014	0.012	8.1	1 in more than 10,000.
4-oz.....	.097	.032	.027	3.59	1 in 65.
8-oz.....	.097	.023	.019	5.10	1 in 1,666.
1-lb.....	.106	.032	.027	3.92	1 in 122.
3-lb.....	.148	.120	.101	1.46	1 in 3.
5-lb.....	.209	.005	.004	52.2	1 in more than 10,000.
50-lb.....	3.57	.70	.59	6.0	1 in more than 10,000.

With the exception of 3-pound packages, on which only one set of data was obtained, and 4-ounce packages where there is apparently a comparatively abnormal result, the chances in the last column of the table would indicate that the calculated maximum error will seldom if ever occur in actual practice.

In the case of individual weights, x is the calculated maximum error on single packages. Since x is calculated from the aim, r is 0.8453 times the average deviation from the aim. This average deviation is

¹ Merriman. Loc. cit., p. 221.

the combination of the average deviation from 0 average minus aim (a) and the average deviation of single packages from the average weight (b), according to the law of propagation of errors, that is, $\sqrt{(a)^2 + (b)^2}$, where (a) and (b) are constituent errors. The resulting chances are given in Table 9.

With the exception of 3-pound and 5-pound packages, on which the data are few, the chances in the last column would indicate that the calculated maximum error will seldom if ever occur in actual practice.

The averages of the chances given in Tables 8 and 9 are 1 in 4,551 and 1 in 2,167, respectively, which, compared to 1 in 1,000, indicate that if the packer aims to make his packages meet the declared weight it is reasonably certain that neither his average nor his individual weights will vary from the declared weight by more than the calculated maxima.

TABLE 9.—*Chances of occurrence of variations greater than the calculated maximum error on single packages.*

Size of package.	Calculated maximum error on single packages (z).	Average deviation from 0 average—aim (a).	Average deviation of single packages from average weight (b).	$0.8453 \times \sqrt{(a)^2 + (b)^2}$ (f).	$\frac{x}{f}$ (f).	Chance of occurrence of variations greater than calculated maximum, based on observed data.
	Ozs.	Oz	Oz.	Oz.		
2-oz.....	0.150	0.014	0.030	0.028	5.35	1 in 3,333.
4-oz.....	.150	.032	.023	.033	4.55	1 in 476.
8-oz.....	.150	.023	.042	.041	3.66	1 in 74.
1-lb.....	.213	.032	.039	.043	4.95	1 in 1,250.
3-lb.....	.363	.120	.094	.130	2.80	1 in 17.
5-lb.....	.409	.005	.169	.144	2.83	1 in 18.
50-lb.....	7.73	.70	.40	.69	11.2	1 in more than 10,000.

The fact that the calculated and observed results check as well as they do on the sizes on which experimental results were obtained justifies a confidence in the calculated figures. In the absence of a large amount of carefully taken experimental results, it is believed that the figures in the last two columns of Tables 5 and 6, "Calculated maximum error on a single package," and "Calculated maximum error on the average of a representative sample," represent a fair approximation to the limiting error of good commercial practice as outlined in this bulletin when applied to tea, coffee, cocoa, spices, and similar granular, free-flowing products.

HAND WEIGHING VERSUS MACHINE WEIGHING.

The exceptionally important part which machine weighing plays in the development of the package-food industry demands that the possibilities of machine weighing be especially considered as a method

which might be as effective as the specified good commercial practice outlined in this bulletin (p. 3). Of the 449 machines at the 126 factories inspected a dozen or more makes were found, no one being exceptionally prevalent. The relative distribution of the five makes most often found was 26, 15, 13, 12, and 11 per cent. The limited number of some makes can without doubt be attributed to the relatively short time they have been on the market.

The claims made for some of these machines by the manufacturers, as taken from their literature, are summarized in Table 10.

Few machines having a capacity in excess of 5 pounds are listed, indicating a predominant demand for the smaller sizes. Table 10 shows also that nearly all manufacturers have different machines for packing different types of products, such as tea, cocoa, and rice.

TABLE 10.—*Claims made by automatic weighing machine manufacturers.*

Maker No.	Type. ¹	Speed.		Accuracy.	Capacity.	Products packed.
		Packages per hour.	Devices for regulating.			
1	A	1,500-2,100	14.....	90 per cent perfect; 100 per cent within $\frac{1}{16}$ ounce.	Ozs. Lbs. 2½ to 6	Rice, coffee, tapioca, sago, split and whole peas, beans, etc.
	B	1,500-2,100	20.....	do.....	2 to 2	Tea, pulverized coffee, rolled oats, flaked corn, wheat, etc.
	C	900-1,500	20.....	do.....	2 to 1	Cocoa, powdered sugar, ground spices.
	D	300-1,200	Several	75 per cent perfect; 100 per cent within $\frac{1}{8}$ ounce.	2 to 2	Cocoa and malted food.
2	A	1,200	None....	100 per cent within $\frac{1}{16}$ ounce.	10 to 2½	Sugar, coffee, spice, snuff, cereals, grain, malt, etc.
	B	1,080	do.....	do.....	1½-2½	Do.
	C	1,080	do.....	do.....	2-5	Do.
	D	720-2,100	do.....	do.....	4 to 1½	Do.
	E	720-2,700	do.....	do.....	1-5	Do.
	F	1,500-2,700	do.....	90 per cent perfect; 100 per cent within $\frac{1}{16}$ ounce.	1 to 1½	Powdered materials such as cocoa, rolled oats, etc.
3	A	1,800	None....	100 per cent balanced weight.	0-1	Coffee, rice, seeds, granulated sugar, cereals, tapioca, salt, etc.
	B	1,800	do.....	do.....	3-8	Do.
	C	1,200-2,100	do.....	do.....	4 to 1	Tea and flaked products.
	D	1,200-2,100	do.....	do.....	4 to 3	Granular products such as cocoa.
	E	1,200-3,600	do.....	do.....	4 to 4	Drugs, cocoa, meals, spices, flours, etc.
4	A	600-2,100	Several	100 per cent within $\frac{1}{16}$ ounce.	8 to 6	Coffee, rice, sugar, peas, beans, tapioca, corn meal, sago, hominy, etc.
	B	600-2,100	do.....	do.....	1-12	Do.
	C	900	do.....	Average variation, 4 ounces on 100 pounds.	1-3	Rolled oats, cereals.
	D	900	do.....	do.....	3-5	Rolled oats, cereals, etc.
5	A	600	do.....	100 per cent within $\frac{1}{16}$ ounce.	4-8	Free-flowing materials, such as coffee, rice, sugar, and salt.
	B	600	do.....	do.....	1	Do.
	C	600	do.....	do.....	2	Do.

¹ These type letters are not those used in the catalogues.

² Several other machines of the same general type, with capacities up to 400 pounds, are listed.

³ Several other similar machines are listed. Machines are made up to 2,400-pound capacity.

TABLE 10.—*Claims made by automatic weighing machine manufacturers—Continued.*

Maker No.	Type.	Speed.		Accuracy.	Capacity.	Products packed.
		Packages per hour.	Devices for regulating.			
5	D	600	Several	100 per cent within $\frac{1}{16}$ ounce.	Ozs. Lbs. 5	Free-flowing materials, such as coffee, rice, sugar, and salt.
	E	600	...do....	...do....	10	Do.
	F	600	...do....	...do....	4 to 10	Cohesive materials, such as cocoa; will also handle tea.
	G	600	...do....	No claim.....	1-8	Cocoa. Not a weighing machine but sometimes used as such.
6	A	1,200	None....	100 per cent within $\frac{1}{16}$ ounce.	4 to 2½	Free-flowing materials, such as coffee, rice, tapioca, salt, and sugar.
	B	1,200	...do....	...do....	1 to 5	Ground spices, baking powder, rolled oats, flour, tea, etc.
7	A	800-1,100	...do....	...do....	2 to 1	Cocoa, baking powder, ground spices, tea, cereals, etc.
	B	800-1,100	...do....	...do....	2 to 1	Sugar, jelly powder, coffee, rice, etc.
18	A	1,500-1,680	Some...	75 per cent absolutely accurate; 100 per cent within $\frac{1}{16}$ ounce.	2 to 7	All free-flowing materials, such as salt, sugar, coffee, farina, cereals, rice, cocoa, corn starch, etc.
	B	1,500-1,800	...do....	...do....	2 to 2	
	C	720-1,800	...do....	...do....	1-15	

⁴ Other sizes up to 25-pound capacity are listed.

The almost unanimous claim of one-sixteenth ounce "accuracy" on practically every size of package might seem extravagant. A wide range of speed is claimed for individual machines, and in some cases devices for regulating it are attached. In the absence of such devices, the regulation is made before the machine is set up. These machines have two essential parts in common, namely a scale, and a device for cutting off or diverting a stream of the product which falls on the scale pan. As a rule, the speed of packing is regulated by a device which controls the size of the stream. The manufacturers of machines seem to have recognized and several have stated that the impact of a large stream upon the scale pan is not conducive to accuracy. Apparently almost any desired accuracy could be obtained if the cross section of the falling stream were made small enough. This view would indicate that the manufacturer's claim for accuracy is not necessarily extravagant, provided the proper speed is maintained. This speed, however, may not be a practicable one.

EXPERIMENTAL WORK ON MACHINES.

A number of commercially operated machines were tested in order to determine the effect of various factors on accuracy, as well as their ability to meet the accuracy of hand weighing. Each machine was examined to make sure that it was in good repair. The operator

was requested to adjust it carefully by his usual method and pack 50 packages. His true aim was worked out as carefully as possible, eliminating scale errors, his speed of packing taken, and the 50 packages accurately reweighed.

TABLE 11.—*Variations in weight of different sizes and of different products packed by machine.*

Experiment No.	Make of machine.	Product.	Declared weight.	Speed.	Average deviation of single packages from average weight.
			<i>Ounces.</i>	<i>Pkgs. per hour.</i>	
860x.....	2	Pepper.....	$\frac{3}{4}$	1,680	.049
853x.....	2	do.....	$\frac{2}{4}$		.034
853y.....	2	do.....	4	(1)	.039
853z.....	2	do.....	4	2,118	.042
		Average.....			.041
862x.....	7	Cocoa.....	1.6	947	.041
21.....	7	do.....	3.2		.032
			<i>Pounds.</i>		
862y.....	7	do.....	1	563	2.211
862z.....	7	do.....	1	622	2.322
		Average.....			.037
857x.....	8	Rice.....	1	2,100	.047
857y.....	8	do.....	2.5	1,500	.074
		Average.....			3.047

¹ Speed lost, but slower than 853z.

² The container was apparently too small, as a quantity of cocoa was spilled in packing after the proper amount had been weighed out by the machine. These results are not included in the average.

³ Average deviation of 2½-pound packages omitted from averages.

The data taken to show the variation in accuracy due to packing different sizes of the same product on the same machine and that due to packing different products are given in Table 11.

With the exception of the two sets of data in which the product was carelessly spilled, very little change of average deviation is noted as the declared weights increase to 1 pound. Neither is any appreciable difference in the averages of the various products noted. The average deviations increase in the following order, namely, cocoa, pepper, rice. The apparent and claimed difficulties in packing the products indicate the reverse order. Numerous ingenious tamping, packing, and stirring attachments which were rendering valuable assistance in the reduction of average deviations were found in use. These facts indicate that the machine manufacturer is successfully meeting the difficulties due to the peculiar nature of the various products.

The data taken to show the effect of speed on accuracy and the possibility of meeting the errors of commercial hand weighing by machines are given in Table 12.

Each group represents data taken on a single machine, all conditions being the same except speed. Each declared weight is 1 pound, except in the case of experiments 856x and 856y, where 12-ounce packages were taken. Since the machine was reset for each speed, the results in column 5 indicate the possible accuracy in setting.

TABLE 12.—*Effect of speed on accuracy of machines.*

Experiment No.	Make of machine.	Aim.	Average weight.	Average minus aim.	Average deviation of single packages from average.	Speed.	Product.
		<i>Ozs.</i>	<i>Ozs.</i>	<i>Oz.</i>	<i>Oz.</i>	<i>Pkgs. per hour.</i>	
854y.....	1	16.09	16.21	0.12	0.029	450	Ground coffee.
854z.....	1	16.09	16.21	.12	.059	679	Do.
854x.....	1	16.09	16.09	.00	.028	783	Do.
Average.....				.08	.039	637	
Average deviation from 0 average—aim.....					.080		
857z.....	2	16.13	16.12	— .01	.076	1,860	Whole coffee.
857u.....	2	16.13	16.33	.20	.106	2,580	Do.
Average.....				.10	.091	2,220	
Average deviation from 0 average—aim.....					.105		
863y.....	3	16.01	15.93	— .08	.052	706	Do.
863x.....	3	16.01	15.98	— .03	.045	1,241	Do.
863u.....	3	16.01	16.02	.01	.048	1,274	Do.
863z.....	3	16.01	15.95	— .06	.059	1,440	Do.
Average.....				— .04	.051	1,165	
Average deviation from 0 average—aim.....					.045		
858z.....	6	16.00	16.06	.06	.063	409	Do.
858y.....	6	16.00	15.99	— .01	.057	818	Do.
858x.....	6	16.00	16.00	.00	.063	1,353	Do.
Average.....				.02	.061	853	
Average deviation from 0 average—aim.....					.023		
856x.....	8	12.15	12.10	— .05	.050	430	Cocoa.
856y.....	8	12.15	12.13	— .02	.121	514	Do.
Average.....				— .03	.085	472	
Average deviation from 0 average—aim.....					.035		

The relation of speed to accuracy is shown graphically in Figure 1. It will be noted that in every case the accuracy increases as the speed decreases. Consequently large variations may be introduced by packing too fast, and, conversely, lower speeds tend to produce smaller variations.

Calculations have been made and data collected which show that variations of 0.106 ounce on the average of 50 packages and 0.213 ounce on single 1-pound packages are approximately the limiting variations of good commercial practice by hand weighing. Computations based on the figures in Table 12 show that when the packages are packed by the various machines the chances of occurrence of variations greater than 0.106 and 0.213 ounce are as given in Table 13.

Since to meet the practical limits of a normal frequency distribution the variation must be of such magnitude that the chances thus calculated should be at least 1 in 1,000, the figures in Table 13 do not indicate that machines weigh as well as hand weighers. All these machines, however, were set by the operator's own method, which consisted in testing four or five individual packages. Such a procedure is based upon the assumption that each package is very near the average, when, as a matter of fact, wide variation occurs, and erroneous settings result. A more accurate setting would be obtained

in each case if four or five packages were weighed together. The average error in determining the true average by the operator's method is the average deviation of single packages, while the average error by weighing five together is $\frac{\text{Average deviation}}{\sqrt{5}}$. Recalculating the chances on this basis, the results shown in Table 14 are obtained.

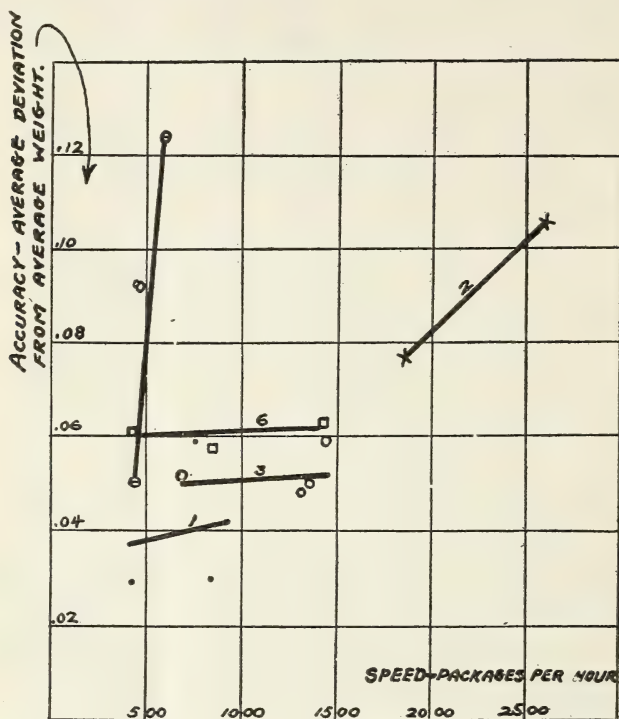


FIG. 1.

RELATION OF SPEED TO ACCURACY OF PACKING BY MACHINE.

KEY • MAKE OF MACHINE #1.
 X MAKE OF MACHINE #2.
 ○ MAKE OF MACHINE #3.
 □ MAKE OF MACHINE #6.
 ⊙ MAKE OF MACHINE #8.

These chances compare more favorably with 1 in 1,000 and indicate that an appreciable error can be eliminated by a little more careful supervision of the machine.

While the errors under average conditions of speed and an erroneous method of setting are not as small as might be expected, the possibilities for increase in accuracy by reduction of speed and proper methods of setting leave little doubt that machines could meet the calculated maximum errors of hand weighing without any appreciable difficulty.

TABLE 13.—*Calculated chance of occurrence of variations of different machines.*

Make of machine.	Chance of occurrence of variations greater than 0.106 ounce on the average weight.	Chance of occurrence of variations greater than 0.213 ounce on single packages.
1	1 in 3.5	1 in 18
2	1 in 2.5	1 in 4.5
3	1 in 17	1 in 83
6	1 in 5,000	1 in 111
8	1 in 59	1 in 15

TABLE 14.—*Recalculated chance of occurrence of variations of different machines.*

Make of machine.	Chance of occurrence of variations greater than 0.106 ounce on the average weight.	Chance of occurrence of variations greater than 0.213 ounce on single packages.
1	1 in more than 10,000	1 in more than 10,000
2	1 in 24	1 in 11
3	1 in 5,000	1 in 500
6	1 in 500	1 in 83
8	1 in 38	1 in 15

PRACTICAL APPLICATION OF EXPERIMENTAL RESULTS.

The figures in the last two columns of Tables 5 and 6, "Calculated maximum error on a single package" and "Calculated maximum error on the average of a representative sample," are of practical interest to the food official and the food packer. They represent the limiting variations in weight given in this bulletin which result from the use of the specified good commercial practice. Since in actual practice the method of packing can not always be studied carefully, the results of check weighings must necessarily be used to determine whether or not the method in use is a good one or is accurately operated.

When the variation in the weight of packages of granular, free-flowing products, such as coffee, spices, sugar, or rice, from the intended weight or aim is greater than the calculated errors given in the tables, it can be inferred that the packages were not packed by as good a method as that outlined in this bulletin as good commercial practice (p. 3).

For making comparisons, weights should be taken net on 50 freshly packed packages and the variations from the aim should be used. When food packers desire to check the efficiency of a method of packing, factory studies must be made to determine the aim. When food officials desire to determine whether short weighing is being practiced, the declared weight must necessarily be used as the aim, since that is the figure which is being checked. If other than freshly packed packages are weighed, allowance must be made for shrinkage.

SUMMARY.

1. The following specifications have been selected to represent good commercial practice in packaging granular, solid, free-flowing food products:

1. That packages of 5 pounds and below be weighed net; larger packages may be weighed gross and tare.

2. That the following types of scales be used—

For packages of 5 pounds and below: Even arm balance, 1-pound beam, $\frac{1}{4}$ -ounce graduation.

For packages over 5 and under 25 pounds: Even arm balance, 2-pound beam, $\frac{1}{4}$ -ounce graduation.

For packages of 25 pounds and above: Platform scale, 50-pound beam, $\frac{1}{4}$ -pound graduation.

3. That scales meet the tolerances prescribed in Bureau of Standards Circular 61 and be used at 20 per cent or more of their capacity for packages of less than 25 pounds and at 10 per cent or more of their capacity for larger packages. Tare weights may be made on the same scale as the gross weights.

4. That the exact balance be estimated to within one-half of the sensibility reciprocal of the scale and the poise be set to within one-fourth of the minimum graduation on the beam.

2. The maximum errors of good commercial practice given in Table 15 have been calculated by the use of methods of least squares, the Bureau of Standards tolerances on scales, and the specifications of good commercial practice.

TABLE 15.—*Calculated maximum errors of good commercial practice.*

Size of package.	For net weighing.		For gross and tare weighing.	
	On single packages.	On the average of a representative sample.	On single packages.	On the average of a representative sample.
	Ounces.	Ounces.	Ounces.	Ounces.
2-ounce.....	0.150	0.097		
3-ounce.....	.150	.097		
4-ounce.....	.150	.097		
8-ounce.....	.150	.097		
1-pound.....	.213	.106		
2-pound.....	.360	.145		
3-pound.....	.363	.148		
4-pound.....	.409	.194		
5-pound.....	.409	.209		
10-pound.....	.544	.373	0.674	0.414
25-pound.....	5.41	2.68	7.49	3.34
50-pound.....	5.41	2.68	7.73	3.57
75-pound.....	5.65	2.92	7.73	3.57
100-pound and above.....	5.65	2.92	8.29	4.13

3. A study of machine weighing has removed practically all doubts of the ability of properly operated machines to meet the calculated maximum errors of hand weighing.

4. Data have been collected which show that the calculated maximum errors represent a close approximation to the maximum variations of good commercial practice.

